Guardbands in Random Testing

Günter Kemnitz

Technische Universität Clausthal
Institut für Informatik
Erzstrasse 1
38678 Clausthal-Zellerfeld
Tel.: +5323 72 2070
Fax.: +5323 72 3572
gkemnitz@informatik.tu-clausthal.de

Abstract

The fault coverage of a random test can be estimated by fault simulation. If the simulation is performed by another random sequence than those used under test or a fault sample is used, a random difference between the simulation result and the fault coverage has to be considered. The simulation result must be larger than the fault coverage that has to be guaranteed. The difference is called guardband. In the paper the distribution of the fault coverage and the distribution of the difference has been derived by the mathematical model of independently detectable faults. Afterwards it is corrected using experimental data. The comparison between theory and experiment unveils a feature of random test, to which no attention has been paid in the past. The correlations in the fault detection process can not be ignored in determining guardbands. As the final result relations for guardband calculation are given.

Key words: test of digital circuits, random test, fault simulation

1 Introduction

The most important parameter of a digital test is the fault coverage. It is the fraction of detectable faults from a set of assumed faults. Using random patterns as stimuli the fault coverage depends mainly on the number of test patterns and less on the test patterns itself. However, it is a random variable.

The paper deals with the following problem: How much fault coverage can be guaranteed if the fault simulation has been performed with other randomly selected input patterns than those used under test? This question is interesting in practice. Random patterns are often used in self-test functions, but also in low cost test systems. Defining the test only by the number of test patterns has a lot of advantages over the alternative, computing, storing and processing a large quantity of exactly defined patterns [1], [2]. For the test of a circuit under operation the input patterns are not known in advance. The fault simulation with an appropriate sample of patterns is the only way to estimate the fault coverage. In many applications it is not enough to know the average fault coverage. The value that can be guaranteed is required. An akin situation arises, if the fault simulation has been done with a sample of faults. Simulation result and fault coverage differ by a random amount and a lower bound has to be guaranteed for the fault coverage.

The term guardband has been taken from analogue testing. Testing a parameter, e.g. a voltage, the measured value must be better than the value that should be guaranteed by the test [3]. The difference, the so called guardband, is necessary to reduce the probability that noise and other disruptions during measuring will cause that bad devices will be classified as good ones. The problem with the fault coverage is akin. The fault coverage should not be lower than a given bound. Otherwise, the number of bad devices classified as good ones will be too large.

A guardband calculation needs the distribution of the parameter under investigation. Section 2 develops a mathematical model to calculate the distribution, the mean value and the variance of the fault coverage out of detection probabilities. Basic features are discussed. For the variance an upper bound has been found. Section 3 describes the guardband problem. Section 4 deals with the guardband size, if the fault coverage is estimated by a fault simulation with other random patterns than those used under test. The necessary size of the guardband for a simulation with a fault sample will be discussed in section 5.

2 Distribution of the fault coverage

The detection probability $p_i(1)$ of a fault i is the probability that the fault will be detected by a single randomly selected input pattern. More detailed and more general explanations the interested reader may find in [1], [2], [4], [5]. To calculate the detection probabilities $p_i(n)$ for a sequence of n input patterns, generally the binomial model is used. A fault will be detected by n input patterns if at least one of the input patterns detect the fault:

$$p_i(n) = 1 - \left(1 - p_i(1)\right)^n \qquad (1)$$

It is based on the assumption that real random patterns are used. It means that patterns may occur by chance multiple times in the sequence. The binomial approach is also a close approximation for a pseudo-random test (no repetition of patterns is possible) if the test set is much shorter than an exhaustive test [6]. Equation (1) can be simplified:

$$p_i(n) = 1 - \mathrm{e}^{-n \cdot q_i} \quad \text{with } q_i = -\ln\left(1 - p_i(1)\right) \geq p_i(1) \qquad (2)$$

p	50%	20%	10%	5%	2%	1%
q	69,315%	22,314%	10,536%	5,129%	2,020%	1,005%

For small detection probabilities it is:

$$p_i(n) = 1 - \mathrm{e}^{-n \cdot p_i(1)} \qquad (3)$$

In the context of guardband calculation the fault coverage is a random variable. By chance it can take values between zero and one. To distinguish the random variable fault coverage from an exactly known fault coverage, the Greek letter ξ will be used.

Now our new idea starts. We introduce auxiliary random variables, one for each fault i that should be one if the fault is detected and zero if it is not detected. The idea behind this is that the fault coverage is the mean value of these auxiliary variables:

$$\xi(n) = \frac{1}{M} \cdot \sum_{i=1}^{M} \zeta_i(n) \qquad (4)$$

(M - number of assumed faults).

The distribution of each of the auxiliary variables $\zeta_i(n)$ is:

$$\begin{aligned} P(\zeta_i(n) = 0) &= 1 - p_i(n) && \text{fault undetectable} \\ P(\zeta_i(n) = 1) &= p_i(n) && \text{fault detectable} \end{aligned} \qquad (5)$$

Their mean values are equal to the detection probabilities:

$$E(\zeta_i(n)) = p_i(n) \tag{6}$$

The variance is:

$$D^2(\zeta_i(n)) = (1 - p_i(n)) \cdot p_i(n) \tag{7}$$

The following presupposes that the faults in the circuit are detected independently of each other. Properly speaking, it is not true. Many faults share control and observation conditions. Resulting from that, the same logical values at least at a part of the inputs are eligible for fault detection. On the other hand, it would not be possible to calculate the distribution of the fault coverage without this assumption. Additional probabilities would be needed of the kind: probability that fault *i* is detectable if fault *j* is (un)detectable. Those data are not available. Therefore, first the model of independently detectable faults is used. Second, the resulting equations are verified by experiments.

Distribution

Under the assumption of independently detectable faults all variations of detectable and undetectable faults have to be compiled. The probability of each variation has to be determined. For all variations with a certain number of detectable faults the probabilities have to be added up. However, because of the exponential growth of the number of variations this approach is only good for a small number of faults.

Figure 1 shows a better algorithm. Taking the distribution of $M = i$ faults and the detection probability of fault $i + 1$ the distribution of $M = i + 1$ faults is calculated. The starting distribution is that of the auxiliary random variable for the first fault $\zeta_1(n)$ with the realizations zero and one. From this the distribution of the first and the second fault

$$P\left(\frac{\zeta_1(n) + \zeta_2(n)}{2} = \frac{m}{2}\right) \text{ with the realizations } \frac{m}{2} \in \left\{0, \quad \tfrac{1}{2}, \quad 1\right\}$$

is calculated, ... The Calculation time of this algorithm grows only with the square of the number of faults.

$P(\xi_1(n) = 0) = 1 - p_1(n)$

$P(\xi_1(n) = 1) = p_1(n)$

DO for $i = 2$ to M

$P(\xi_i(n) = 0) = P(\xi_{i-1}(n) = 0) \cdot (1 - p_i(n))$

$P(\xi_i(n) = 1) = P(\xi_{i-1}(n) = 1) \cdot p_i(n)$

DO for $m = 1$ to $i - 1$

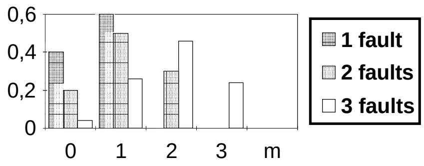
$$P\left(\xi_i(n) = \frac{m}{i}\right) = P\left(\xi_{i-1}(n) = \frac{m}{i-1}\right) \cdot (1 - p_i(n)) + P\left(\xi_{i-1}(n) = \frac{m-1}{i-1}\right) \cdot p_i(n)$$

OD

OD

<u>Distribution for 1 to 3 faults</u>

$p_1(n) = 0{,}6$ | $p_2(n) = 0{,}5$ | $p_3(n) = 0{,}6$



<u>Distribution for 6, 14 and 26 faults</u>

$p_i(n)$: 2× 0,5; 0,6; 0,7; 2×0,8 | 3×0,4; 3×0,7; 0,8; 0,9 | 0,4; 3×0,5; 6×0,8; 2×0,9

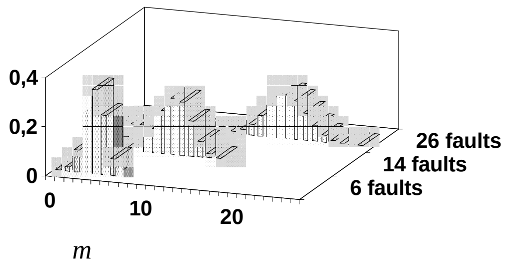


Figure 1: Algorithm to calculate the distribution of the fault coverage with examples ($\xi_i(n)$ - distribution of the fault coverage for i faults; $p_i(n)$ - detection probability of fault i; m - number of detectable faults; M - number of assumed faults; n - number of test patterns)

The example in Figure 1 shows that the fault coverage converges to a normal distribution with a growing number of faults. A proof, presupposing independently detectable faults, can be found in [7]. Figure 2 shows the fault coverage of a larger combinational circuit with stuck-at faults. It is also nearly normal distributed.

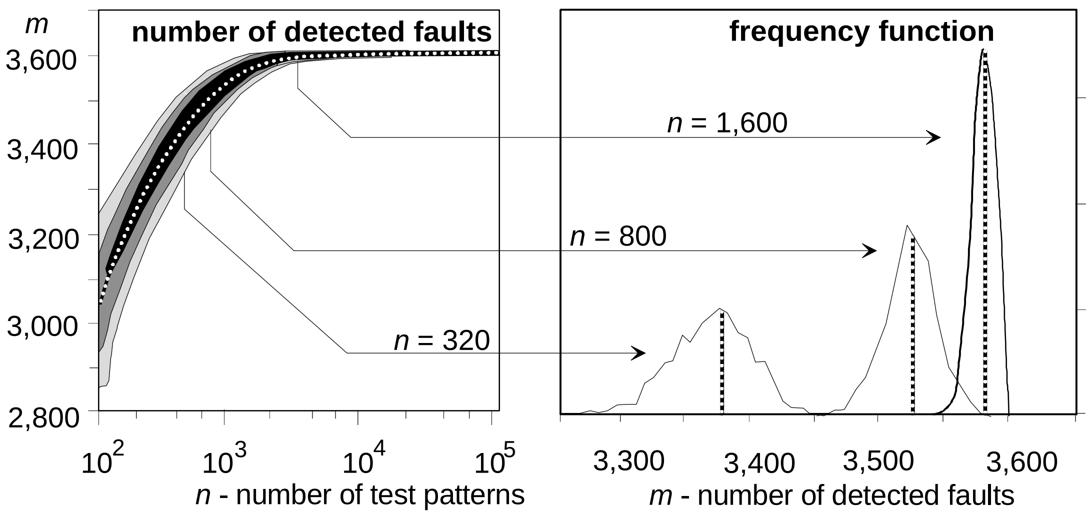


Figure 2: Fault coverage of the benchmark circuit c3540 [8] (result of a fault simulation with 1,000 different random sequences; number of faults $M = 3{,}605$; redundant faults are removed from fault list and equivalent faults are united to one fault)

Mean value

The mean value of a sum of independent random variables is the sum of the mean values of the summands. The fault coverage is the mean value of the auxiliary variables $\zeta_i(n)$. So, the sum has still to be divided by the number of assumed faults.

$$E(\xi(n)) = \frac{1}{M} \cdot \sum_{i=1}^{M} p_i(n) \approx 1 - \frac{1}{M} \sum_{i=1}^{M} e^{-n \cdot p_i(1)} \qquad (8)$$

Equation (8) allows the following conclusions for the average fault coverage. As for the single detection probabilities, it converges to one with a growing number of test patterns. The detection probabilities in a circuit vary by some orders of magnitude. As usual, the majority of faults is easily detectable. They will be found with a few hundred random patterns. The fault coverage grows fast at the begin of the test, reaching about 80% to 90%. A small fraction of the faults is hard to detect. To cover the last few percents of faults may cost an increase in the test length by multiple orders of magnitude.

In practice, an interesting question is whether the fault coverage or the required test length can be estimated by a much shorter test sequence than those used under test. It would allow to save a huge amount of simulation time. Equation (8) shows that it is impossible. For short test sequences the growth of the fault coverage depends almost exclusively on the detection probabilities of easy to detect faults. The effect of the hard to detect faults is small in comparison to the variations of the simulation result. Not even conclusions about the order of magnitude of the detection probabilities for the hard to detect faults can be drawn. Without this information it is impossible to predict the required test length for a fault coverage higher than the simulation result or for the coverage of a much longer test set. The fault simulation to estimate the fault coverage has to be performed with the same number of test patterns as planed for the test.

Variance

The variance of the sum of independent random variables is the sum of the variances of the summands. For the fault coverage is the mean value of the auxiliary variables $\zeta_i(n)$, it is:

$$D^2(\xi(n)) = \frac{1}{M^2} \cdot \sum_{i=1}^{M} p_i(n) \cdot (1 - p_i(n)) \approx \frac{1}{M^2} \cdot \sum_{i=1}^{M} e^{-n \cdot p_i(1)} \cdot \left(1 - e^{-n \cdot p_i(1)}\right) \qquad (9)$$

It converges to zero with a growing number of faults. If the test set is very long, the majority of faults has detection probabilities close to one ($n \cdot p_i >> 1$). The number of summands contributing in equation (9) to the variance becomes smaller with the test length and so also the variance.

Without knowing the single detection probabilities an upper bound of the variance can be given. When all detection probabilities are equal, it has a maximum for a given mean value and a given number of faults. The distribution of the maximum is a binomial distribution. The proof is given in the appendix.

$$D^2(\xi(n)) \leq \frac{1}{M} \cdot E(\xi(n)) \cdot (1 - E(\xi(n))) \tag{10}$$

This unequation allows to measure the effect of the correlations in the fault detection process. Let us introduce a parameter ε:

$$\varepsilon = \frac{M \cdot D^2(\xi(n))}{E(\xi(n)) \cdot (1 - E(\xi(n)))} \tag{11}$$

According to equation (10) ε can not be larger than one for uncorrelated faults. However, it can exceed the bound if interdependencies in the detection process exist. The following will illustrate this. Let us assume that the number of faults for a given circuit has been doubled by listing or counting each fault two times. This corresponds to a fault set with multiple pairs of equivalent faults or faults that will be detected always simultaneously. The trick will neither change the mean value of the fault coverage nor the variance. Only the number of assumed faults M doubles. And so ε becomes two times as large as for a set of independent faults. Obviously, by a further increase of the number of equivalent faults, ε may become larger than one.

Usually, equivalent faults are removed from the fault list before fault simulation. From each class of equivalent faults only one is taken. This has also be done with the fault list used to produce Figure 2. But there are other dependencies. The control and observation paths are similar for many faults. So, faults require partly the same input patterns and will be detected often by the same randomly selected test pattern. Table 1 column 4 shows the values of ε for the experiment in Figure 2. Although, equivalences have been removed, the numbers look as if on average up to 5 and more faults would have been detected in each random sequence with the same pattern.

n	fault simulation with all 3,606 stuck-at faults			fault simulation with a sample of 1,000 faults			fault simulation with a sample of 300 faults		
	$E(\xi(n))$	$\sqrt{D^2(\xi(n))}$	ε	$E(\xi(n))$	$\sqrt{D^2(\xi(n))}$	ε	$E(\xi(n))$	$\sqrt{D^2(\xi(n))}$	ε
160	88.5%	1.28%	5.8	88.2%	1.41%	1.9	89.6%	1.87%	1.1
320	93.5%	0.88%	4.6	93.2%	1.04%	1.7	94.6%	1.42%	1.2
800	97.6%	0.48%	3.5	97.5%	0.63%	1.6	98.4%	0.76%	1.1
1,600	99.2%	0.20%	1.8	99.2%	0.28%	1.0	99.7%	0.36%	1.2
3,200	99.7%	0.08%	0.8	99.7%	0.11%	0.4	99.9%	0.11%	1.0
6,400	99.8%	0.05%	0.5	99.8%	0.08%	0.4	100%	0	--

Table 1: The parameter ε for a complete stuck-at fault set and for two fault samples (circuit c3540 [8], mean value and variance have been estimated by fault simulations with 1,000 different random sequences)

With a growing test length and a growing fault coverage the interdependencies decrease. All faults with high detection probabilities are detected almost by each random sequence of the corresponding test length. They do not contribute to the variance. Between the harder to detect faults are probably also some interdependencies left. But the effect of the safely detectable faults outweighs them.

Interdependencies in the fault detection process do not only increase the variance. They can also change the shape of the distribution. The reason for it is a large group of faults detectable by the same or almost the same set of input patterns. Let us assume a circuit with 10 faults, where 8 faults will be detected always simultaneously. Thus, the number of detectable faults is limited to $m \in \{0,\ 1,\ 2,\ 8,\ 9,\ 10\}$. The distribution of the fault coverage is divided in two ranges. Figure 3 shows this effect for a real circuit with stuck-at faults. From the position and the distance of the ranges it can be concluded that the fault group consists of about 80 faults with a detection probability of about 10^{-5}.

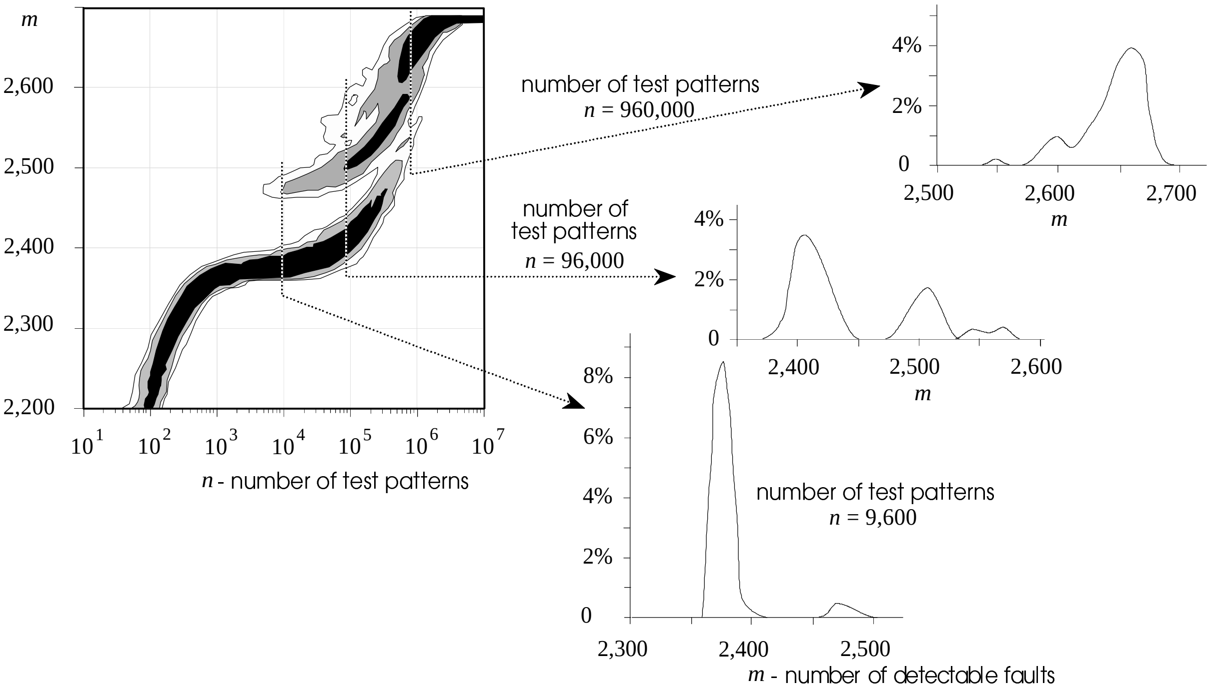


Figure 3: Distribution of the fault coverage of the circuit c2670 [8]. As far as they have been found equivalent faults have been removed from the fault list. Nonetheless, the result looks as if one class of equivalent faults has been forgotten (2685 stuck-at faults; result of a fault simulation with 1,000 different random sequences)

3 Guardbands

The fault coverage of a test set is a quality parameter, manufacturers guarantee for. It must be at least as large as a given lower bound FC_{min} . However, the fault coverage is a random variable which can take a value between zero and one by chance. A lower bound can only be given with a small error probability (typically the fraction of a percent up to some percent):

$$P(\xi < FC_{\min}) < \alpha \qquad (12)$$

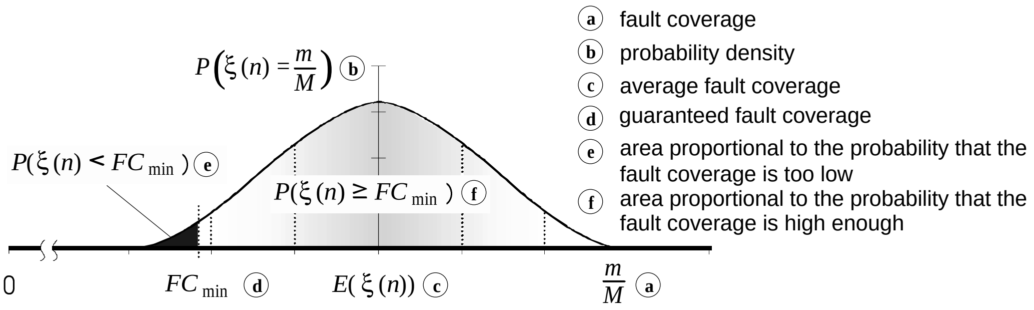


Figure 4: One-side interval estimation of the guaranteed fault coverage

The average fault coverage must be larger than the guaranteed fault coverage by a certain amount called guardband G. As shown in the last section, the fault coverage is often nearly normal distributed. The guardband for a nomal random variable must be about 2 to 4 times larger than the standard deviation:

$$G \geq k \cdot \sqrt{D^2(\xi(n))} \quad \text{with} \quad \alpha = \Phi(-k) \tag{13}$$

(α - error probability; $\Phi(-k)$ - value of the standardized normal distribution).

The standard deviation of the fault coverage can be estimated by equation (10) using the mean value and the parameter ε:

$$G \geq k \cdot \varepsilon \cdot \sqrt{\frac{E(\xi(n)) \cdot (1 - E(\xi(n)))}{M}} \tag{14}$$

The larger the number of faults is the smaller can be the guardband.

The mean value and the variance of the fault coverage $\xi(n)$ has to be estimated. This requires a fault simulation with M_s faults and n random patterns. The result of the fault simulation is the estimated mean value of the fault coverage. It differs from the mean value by a random amount. The fault coverage itself also differs from the mean value by a random amount. So, both random differences have to be considered for the guardband (Figure 5).

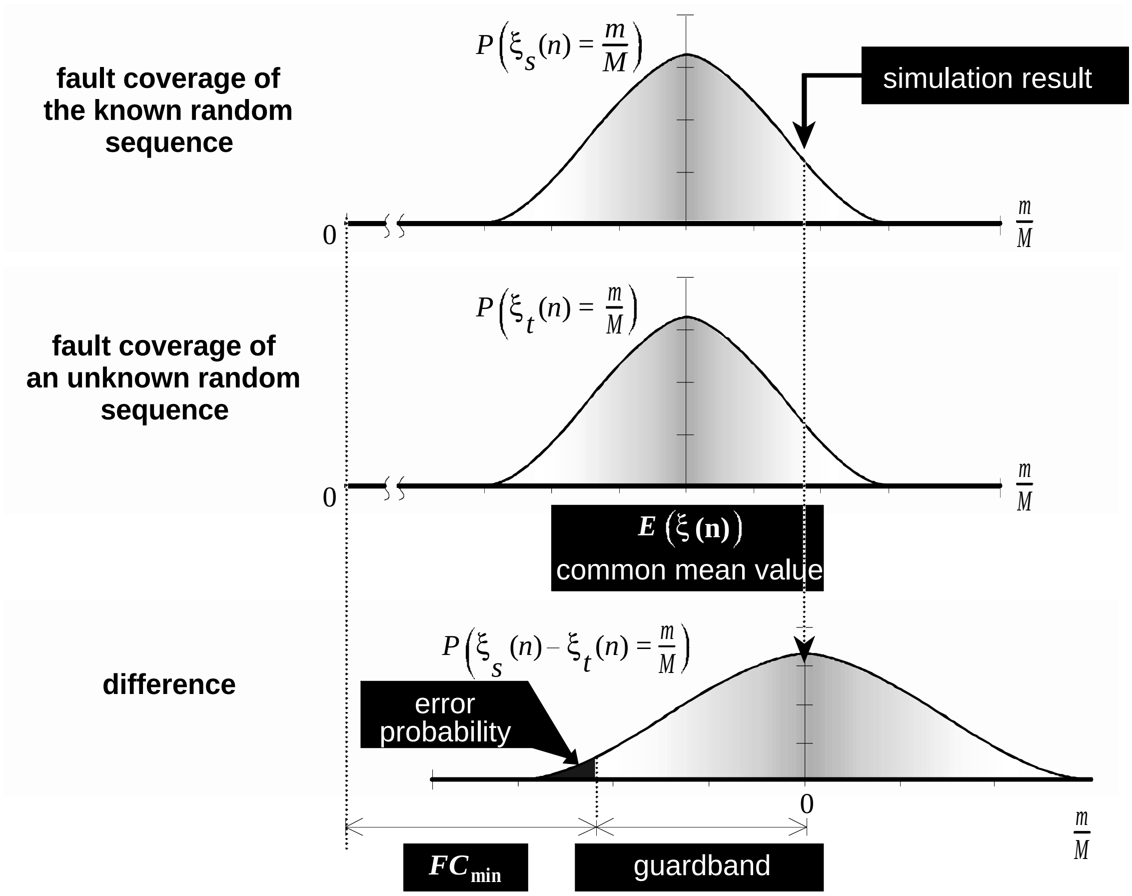


Figure 5: Guardband between the fault coverage of a known random sequence and the value that can be guaranteed for an arbitrary random sequence

4 Simulation with another random sequence

The fault coverage will be determined by another random sequence than those used under test. The number of random patterns should be the same. In this case, the simulation result and the fault coverage are two independent random variables with the same distribution. The variance of the sum of two independent random variables is the sum of the variances. It doubles. The standard deviation as the square root of the variance increases by the factor $\sqrt{2}$. By the same amount the guardband has to be increased in comparison to equation (14) where the mean value has been assumed to be exactly known:

$$G_\Delta \geq \sqrt{2} \cdot k \cdot \varepsilon \cdot \sqrt{\frac{E(\xi(n)) \cdot (1 - E(\xi(n)))}{M}} \qquad (15)$$

The required guardband can be reduced by repeating the fault simulation with z different random sequences of length n. The estimated mean value becomes the mean value of the simulation results. The necessary size of the guardband reduces down to:

$$G_\Delta \geq \sqrt{1 + \frac{1}{z}} \cdot k \cdot \varepsilon \cdot \sqrt{\frac{E(\xi(n)) \cdot (1 - E(\xi(n)))}{M}} \qquad (16)$$

(Multiple simulation results allow also a more accurate estimation of the variance.)

5 Simulation with a fault sample

The simulation with a fault sample costs less simulation time. In return, the variance of the simulation result is higher. Using the upper bound (10), it could be assumed that the variance $D^2(\xi)$ grows conversely proportional to the reduction of the fault sample. The standard deviation $\sqrt{D^2(\xi)}$ grows with the root. The distribution becomes broader and flatter (Figure 6).

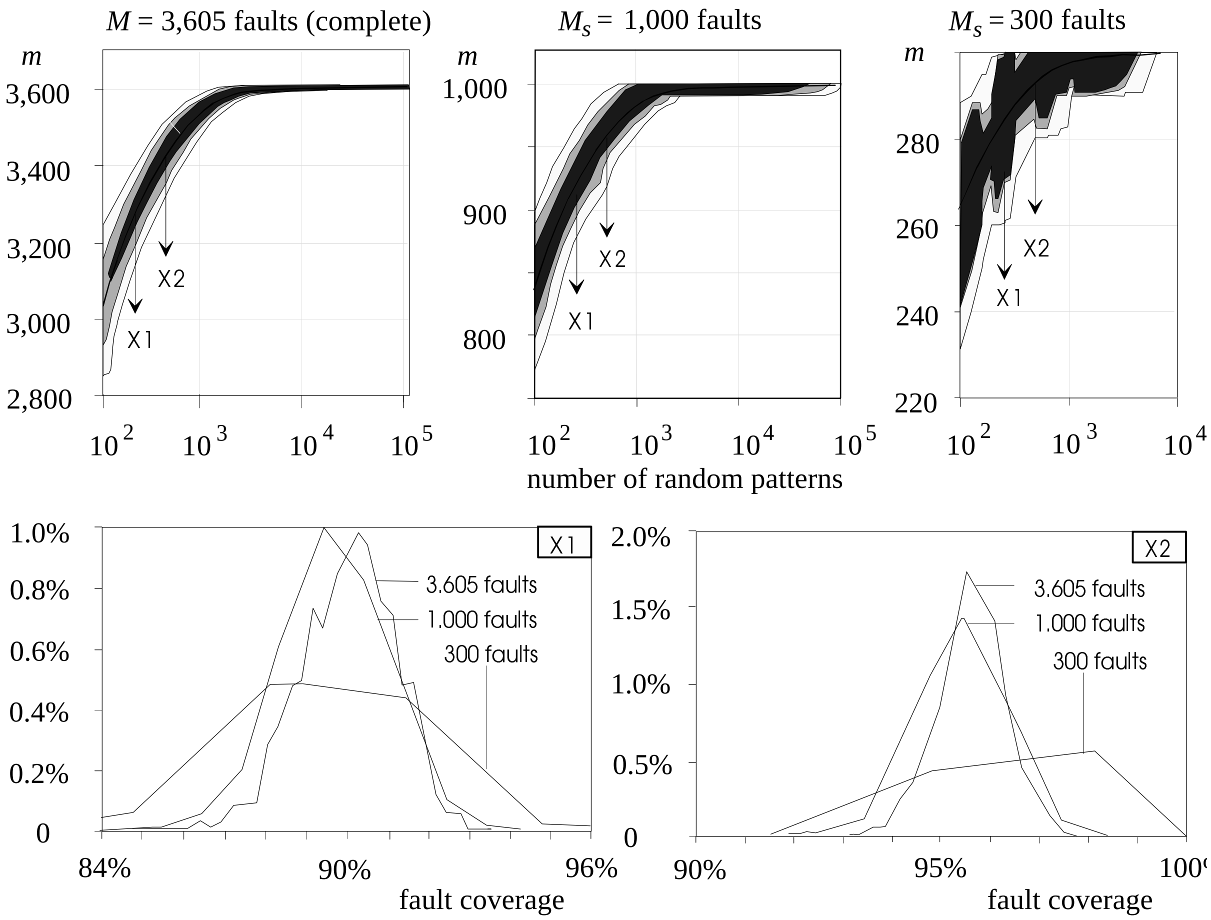


Figure 6: Distribution of the fault coverage of different samples of stuck-at faults (circuit c3540 [8], simulation with 1,000 random sequences per fault set)

Especially for large fault samples, the guardband size depends upon whether the simulation is performed with the same or a different random sequence than the test. Using different random sequences, simulation result and fault coverage are two independent random variables. The variances add:

$$D^2(\xi_s - \xi_t) = D^2(\xi_s) + D^2(\xi_t) \qquad (17)$$

Using the upper bound for the variance, equation (17) becomes:

$$D^2(\xi_s - \xi_t) \le \left(\frac{\varepsilon^2}{M} + \frac{\varepsilon_s^2}{M_s}\right) \cdot E(\xi(n)) \cdot \left(1 - E(\xi(n))\right) \qquad (18)$$

Is the fault simulation performed by the same random patterns as the test, the simulation result is already the exact coverage for M_s of M faults. This fraction does not contribute to the variance of the difference. In comparison to equation (18) the variance of the difference is smaller by the factor $(1-M_s)/M$:

$$D^2(\xi_s-\xi_t)\geq\frac{1-M_s}{M}\left(\frac{\varepsilon^2}{M}+\frac{\varepsilon_s^2}{M_s}\right)\cdot E(\xi(n))\cdot(1-E(\xi(n))) \qquad (19)$$

For a small fault sample $M_s << M$ the variance of the fault coverage can be neglected. The factor $(1-M_s)/M$ (19) is also close to one:

$$D^2(\xi_s-\xi_t)\approx\varepsilon_s^2\cdot\sqrt{\frac{E(\xi(n))\cdot(1-E(\xi(n)))}{M_s}} \qquad (20)$$

The size of the guardband is:

$$G_s\approx k\cdot\varepsilon_s\cdot\sqrt{\frac{E(\xi(n))\cdot(1-E(\xi(n)))}{M_s}} \qquad (21)$$

Equation (21) gives the impression that the guardband must be increased conversely proportional to the root of the number of simulated faults. The real proportions are much better. Table 1 shows also the mean values and the variances of the simulation with fault samples. The increase of the variance is much smaller than it could have been expected according to equation (20). With the reduction of the size of the fault sample the parameter ε also becomes smaller.

The parameter ε has been introduced to quantify the interdependencies in the fault detection process. The more fault assumptions are distributed in a given circuit, the more interdependencies are to be expected and vice versa. The reason is obviously that the number of control and observation paths is limited in a circuit. Many assumed faults must share control and observation conditions. It means that similar input patterns will detect them. The variance does depend less on the total number of faults but more on the number of groups of similar detectable faults. The number of those groups is limited by the circuit structure. Selecting a fault sample reduces mainly the number of similar detectable faults within the groups and not so much the number of groups. So, the variance is less effected. Of cause, this explanation is simplified. Further investigations are required to understand the phenomenon of interdependences in the fault detection process better.

The parameter ε is a measure for the efficiency of a fault simulation. The aim of a fault simulation is a close prediction of the fault coverage. The costs depend on the number of assumed faults. A conclusion from this paper is also that a complete fault simulation does not repay, except if it is performed with the same patterns as the test. The reduction of the standard deviation is out of relation to the number of simulated faults. A fault simulation with a fault sample is more economic. Besides, using the same patterns than the test is a con job. It provides the exact value of the fault coverage for the assumed faults. However, these are not the faults to be found under test. Between the fault coverages of modelled faults and of real faults also a random difference has to be considered. But this is another problem that should not be solved in this paper.

Summary

The fault coverage of a random test is a random variable. It is mostly, but not always normal distributed. The mean value converges with a growing number of test patterns to 100%. But it is not possible to estimate the mean value for a long test sequence by a fault simulation with a much shorter test sequence.

For the variance an upper bound has been found which depends only on the mean value and the number of assumed faults. This bound holds if all assumed faults are detectable independently of each other. Interdependencies in the detection process increase the variance far beyond this bound. So, interdependencies can be quantified and measured by simulation experiments.

A guardband calculation has to take into account two random variables. The result of the fault simulation and the fault coverage differ from the common mean value by a random amount. So, the variances of both random variables have to be added. The final result is that the guardband must be approximately:

$$G = 4\ldots 15 \cdot \sqrt{\frac{\text{simulation\_result} \cdot (1 - \text{simulation\_result})}{\text{number\_of\_simulated\_faults}}}$$

Especially, if a fault coverage close to 100% has to be guaranteed, the required guardband is large in comparison to the allowed difference to 100%. It is, because the guardband reduces only proportional to the root of the term (1-simulation_result).

An interesting conclusion is that the guardbands for fault samples do increase much less than conversely proportional to the root of the number of simulated faults. Reducing the number of faults, the factor before the root also becomes smaller. It is because there are less interdependencies in a fault sample than in a complete fault set. So, it looks to be more efficient to perform the fault simulation for a random test only with a fault sample and not with the whole fault set.

References

[1] Illman, R. J.: Self-tested data flow logic: A new approach. In: IEEE Design and Test of computers, No. 4, 1985, 50-58

[2] Lombardi, F.; Sami, M. (eds): Testing and diagnosis of VLSI and ULSI. Kluwer Academic Publishers, 1988

[3] Williams, R. H.; Hawkins, C. F.: The effect of guardbands on errors in production testing. European Test Conf., Rotterdam, 1993, 2-7

[4] David, R.; Blanchet, G.: About random fault detection of combinational networks. IEEE Transaction on Computers No. 6, 1976, 659-664

[5] Cirit, M. A.: Switch level random pattern testability analysis. DAC-25, 1988, 587-590

[6] Chin, C. K.; McCluskey, E. J.: Test length for pseudo random testing. International Test Conference, 1985, 94-9

[7] Kemnitz, G.: Self test of digital circuits: tools, design approach and software support. Dissertationsschrift, Dresden University of Technology, 1991 (in German)

[8] Brglez, F.; Fujiwara, H.: A neutral netlist of 10 combinatorial benchmark circuits and a target translator in FORTRAN. Int. Symposium on Circuits and Systems; Special Session on ATPG and Fault Simulation, 1985

Appendix

Proof for unequation (10)

Let us substitute all detection probabilities by the sum of the mean value and a difference:

$$p_i(n) = E(\xi(n)) + \delta_i \quad \text{with} \quad E(\xi(n)) = \frac{1}{M} \cdot \sum_{i=1}^{M} p_i(n) \quad \text{and} \quad \sum_{i=1}^{M} \delta_i = 0$$

Inserted in (10):

$$D^2(\xi(n)) = \frac{\sum_{i=1}^{M} \left(E(\xi(n)) + \delta_i\right) \cdot \left(1 - E(\xi(n)) - \delta_i\right)}{M^2}$$

$$\leq \frac{E(\xi(n)) \cdot \left(1 - E(\xi(n))\right)}{M}$$

The simplified unequation $\sum_{i=1}^{M} \delta_i^2 \geq 0$ is true for all δ_i.